LAMONT-DOHERTY GEOLOGICAL OBSERVATORY
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FURTHER INVESTIGATION OF THE INTEGRAL SOLUTION OF THE SOUND FIELD IN MULTILAYERED MEDIA; A LIQUID-SOLID HALF SPACE WITH A SOLID BOTTOM

by

Henry W. Kutschale

CU-6-71

Technical Report No. 6

Contract N00014-67-A-0108-0016 with the Office of Naval Research
(Work done on behalf of U. S. Naval Ordnance Laboratory)

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ABSTRACT

The integral solution of the sound field is derived from a laminated, interbedded liquid-solid half space. The last layer of infinite thickness is solid. The integral over wave number is conveniently transformed into the complex wave number plane yielding a sum of normal modes of propagation plus the sum of integrals along branch cuts. Waves corresponding to the normal modes predominate at ranges beyond several times the water depth and are considered in detail. Sample numerical computations are presented for propagation of Rayleigh waves in shallow water on the Arctic continental shelf.

FIGURES

DEFINITION OF SYMBOLS

α_m ; compressional-wave velocity in the m-th layer

β_m ; shear-wave velocity in the m-th layer

h_m ; thickness of the m-th layer

z ; vertical coordinate

r ; range between source and detector

t ; time

ω ; angular frequency

c ; phase velocity

k ; wave number

i ; $\sqrt{-1}$

Im ; imaginary part of a complex number

ϕ_m ; velocity potential in the m-th layer

$(p_{zz})_m$; normal stress in the m-th layer parallel to z axis

τ_m ; tangential stress in the m-th layer

p_m ; pressure in the m-th liquid layer

u_m ; horizontal particle displacement in the m-th layer in the radial direction

w_m ; vertical particle displacement in the m-th layer

$\dot{u}_m$; horizontal particle velocity in the m-th layer in the radial direction

$\dot{w}_m$; vertical particle velocity in the m-th layer

ρ_m ; density in the m-th layer

ρ_s ; density at the source

J_0 ; Bessel function of order 0
Y_0 ; Y Bessel function of order 0
$H_0^{(1)}$; Hankel function of the first kind of order 0
$H_0^{(2)}$; Hankel function of the second kind of order 0
J_1 ; Bessel function of order 1

$$r_{\alpha_m} = \sqrt{\left(\frac{c}{\alpha_m}\right)^2 - 1}\,, \quad c > \alpha_m$$

$$r_{\alpha_m} = -i\sqrt{1 - \left(\frac{c}{\alpha_m}\right)^2}\,, \quad c < \alpha_m$$

$$r_{\beta_m} = \sqrt{\left(\frac{c}{\beta_m}\right)^2 - 1}\,, \quad c > \beta_m$$

$$r_{\beta_m} = -i\sqrt{1 - \left(\frac{c}{\beta_m}\right)^2}\,, \quad c < \beta_m$$

$$P_m = k h_m r_{\alpha_m}$$

$$Q_m = k h_m r_{\beta_m}$$

$$\gamma_m = 2\left(\frac{\beta_m}{c}\right)^2$$

INTRODUCTION

In a previous investigation (Kutschale, 1970) we derived by matrix methods of Thomson (1950), Haskell (1953), Dorman (1962), and Harkrider (1964) the integral solution of the wave equation for point sources of harmonic waves in a liquid layer of a multilayered half space of interbedded liquid and solid layers. The solution is in a form convenient for computation on a high-speed digital computer. However, only the case was considered of a liquid half space underlying the stack of layers. Many experiments on shallow-water propagation of low-frequency waves (Pekeris, 1948, Tolstoy, 1958, Hunkins and Kutschale, 1961) have shown that this representation of the sediments by a layered liquid bounded below by a liquid half space is often useful and valid, but in some experiments on shallow-water propagation in the Arctic Ocean it has been necessary to consider the rigidity of the sediments to take account of propagation of Rayleigh waves in the layered system. The ice sheet is represented by a solid layer at the surface.

In the present report we extend the integral solutions of pressure and particle displacements derived from a layered medium bounded below by a liquid half-space to a layered medium bounded below by a solid half-space. The integral solutions of pressure or particle displacements are evaluated as a sum of normal modes of propagation plus the sum of branch line integrals in the complex wave number plane. Waves corresponding to the normal modes generally predominate at ranges beyond several times the water depth and will be considered in detail. The branch line integrals are difficult to evaluate numerically for a solid half space underlying the stack of layers and discussion of them is omitted. The formulas for pressure and particle

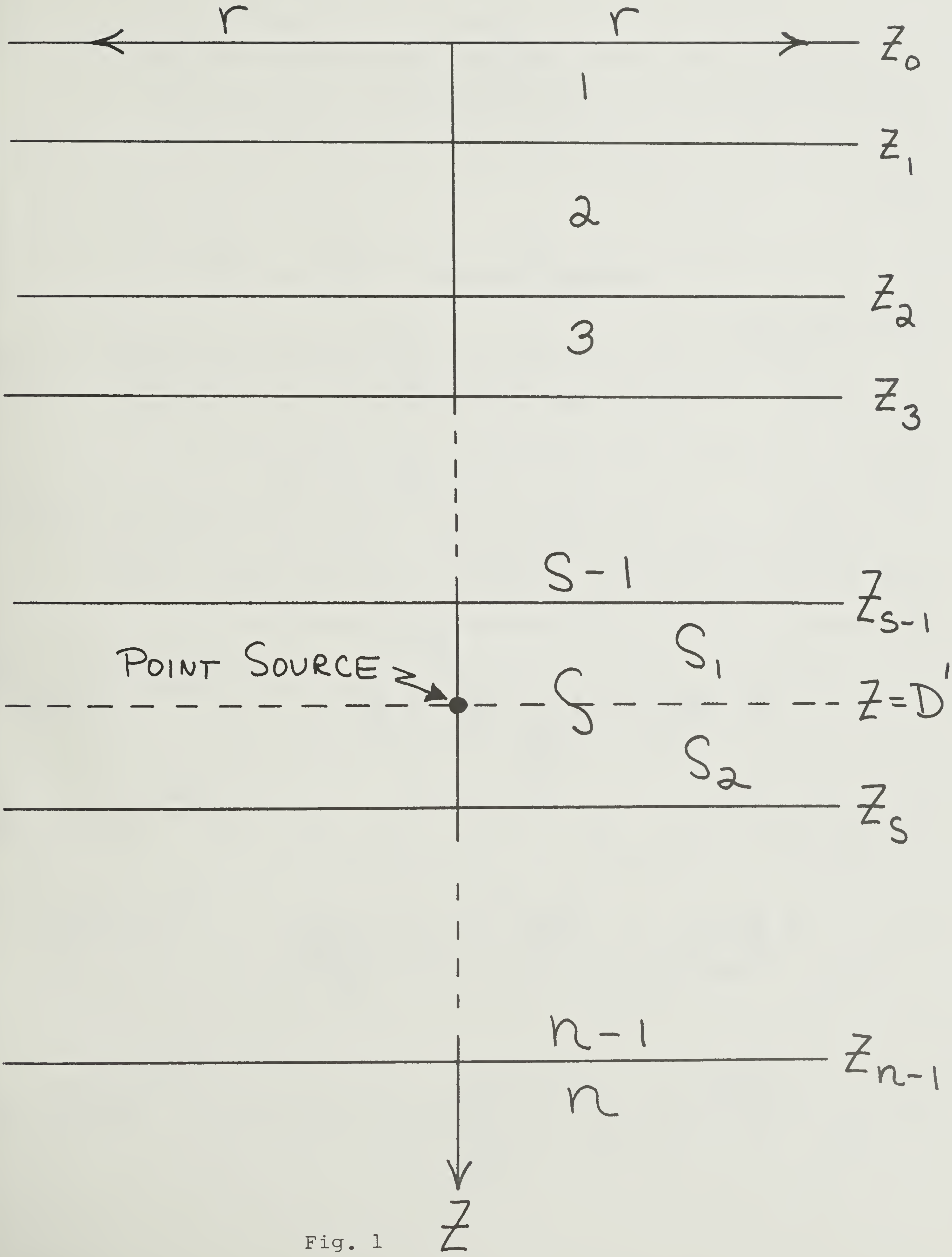


Fig. 1

displacements of the normal modes are incorporated in the same computer programs for the IBM 360/91 used for the case of a liquid half space underlying the stack of layers (Kutschale, 1970).

FORMAL SOLUTION

Source Free Case

Consider the n-layered interbedded liquid-solid half-space shown in Figure 1 in which the last layer of infinite thickness is solid. We assume that guided waves may propagate in the layered system; that is, that c may be less than β_n.

A point source of harmonic waves is located in one of the liquid layers. The velocity potential in the m-th layer, which is assumed to be liquid, satisfies the wave equation

$$\nabla^2 \phi_m - \frac{1}{\alpha_m^2} \ddot{\phi}_m = 0$$

In this layer

$$(p_{zz})_m = -p_m = \rho_m \frac{\partial \phi_m}{\partial t}$$

$$\frac{\dot{w}_m}{c} = \frac{1}{c} \frac{\partial \phi_m}{\partial z}$$

$$\frac{\dot{u}_m}{c} = \frac{1}{c}\frac{\partial \phi_m}{\partial r}$$

Solutions for ϕ_m, $\frac{\dot{w}_m}{c}$, and $(p_{zz})_m$ are given by

$$\phi_m(r,z,t,k) = \int_0^\infty \hat{\phi}_m(z)\, J_0(kr)\, e^{i\omega t}\, dk$$

$$\frac{\dot{w}_m(r,z,t,k)}{c} = \int_0^\infty \frac{\hat{\dot{w}}_m(z)}{c}\, J_0(kr)\, e^{i\omega t}\, dk$$

$$(p_{zz})_m(r,z,t,k) = \int_0^\infty (\hat{p}_{zz})_m(z)\, J_0(kr)\, e^{i\omega t}\, dk$$

where

$$\hat{\phi}_m(z)e^{i\omega t} = \left(A_m e^{ikr_{\alpha_m}z} + A_m' e^{-ikr_{\alpha_m}z}\right)e^{i\omega t}$$

$$\frac{\hat{\dot{w}}_m(z)}{c}e^{i\omega t} = \frac{ikr_{\alpha_m}}{c}\left(A_m e^{ikr_{\alpha_m}z} - A_m' e^{-ikr_{\alpha_m}z}\right)e^{i\omega t}$$

and

$$(\hat{p}_{zz})_m(z)e^{i\omega t} = i\omega\rho_m\left(A_m e^{ikr_{\alpha_m}z} + A_m' e^{-ikr_{\alpha_m}z}\right)e^{i\omega t}$$

The A_m and A_m' are constants.

Placing the origin at the (m-1)-st interface we have

$$\frac{\dot{\hat{w}}_{m-1}}{c} = \frac{ikr_{\alpha_m}}{c}(A_m - A_m') \tag{1}$$

$$(\hat{p}_{zz})_{m-1} = i\omega\rho_m (A_m + A_m')$$

at $z = 0$

and

$$\frac{\dot{\hat{w}}_m}{c} = -\frac{kr_{\alpha_m}}{c}(A_m + A_m')\sin P_m + (A_m - A_m')\frac{ikr_{\alpha_m}}{c}\cos P_m \tag{2}$$

$$(\hat{p}_{zz})_m = (A_m + A_m')\,i\omega\rho_m \cos P_m - (A_m - A_m')\,\omega\rho_m \sin P_m$$

at $z = h_m$

Substituting expressions for $(Am - Am')$ and $(Am + Am')$ from Equations (1) in Equations (2) yields

$$\frac{\hat{\dot{w}}_m}{c} = \frac{\hat{\dot{w}}_{m-1}}{c} \cos P_m + (\hat{p}_{zz})_{m-1} \frac{i r_{\alpha_m}}{\rho_m c^2} \sin P_m$$

$$(\hat{p}_{zz})_m = \frac{\hat{\dot{w}}_{m-1}}{c} \frac{i \rho_m c^2}{r_{\alpha_m}} \sin P_m + (\hat{p}_{zz})_{m-1} \cos P_m$$

or in matrix notation

$$\begin{bmatrix} \dfrac{\hat{\dot{w}}_m}{c} \\ \\ (\hat{p}_{zz})_m \end{bmatrix} = \begin{bmatrix} \cos P_m & \dfrac{i r_{\alpha_m}}{\rho_m c^2} \sin P_m \\ \\ \dfrac{i \rho_m c^2}{r_{\alpha_m}} \sin P_m & \cos P_m \end{bmatrix} \begin{bmatrix} \dfrac{\hat{\dot{w}}_{m-1}}{c} \\ \\ (\hat{p}_{zz})_{m-1} \end{bmatrix}$$

$$= a_m \begin{bmatrix} \dfrac{\hat{\dot{w}}_{m-1}}{c} \\ \\ (\hat{p}_{zz})_{m-1} \end{bmatrix}$$

In general for an n-layered liquid half-space

$$\begin{bmatrix} \dfrac{\dot{\hat{w}}_{n-1}}{c} \\ \\ (\hat{p}_{zz})_{n-1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \dfrac{\dot{\hat{w}}_0}{c} \\ \\ (\hat{p}_{zz})_0 \end{bmatrix}, \tag{2}$$

$$\begin{bmatrix} A_{11} & A_{12} \\ \\ A_{21} & A_{22} \end{bmatrix} = a_{n-1} a_{n-2} \cdots a_1 ,$$

since the vertical particle velocity and pressure are continuous across each interface.

Consider now a solid layer between two liquid layers or at the surface of the laminated half-space. For this layer we may write (Thomson, 1950; Dorman, 1962)

$$\begin{bmatrix} (a_m)_{11} & (a_m)_{12} & (a_m)_{13} & (a_m)_{14} \\ \\ (a_m)_{21} & (a_m)_{22} & (a_m)_{23} & (a_m)_{24} \\ \\ (a_m)_{31} & (a_m)_{32} & (a_m)_{33} & (a_m)_{34} \\ \\ (a_m)_{41} & (a_m)_{42} & (a_m)_{43} & (a_m)_{44} \end{bmatrix} \begin{bmatrix} \dfrac{\dot{\hat{u}}_{m-1}}{c} \\ \\ \dfrac{\dot{\hat{w}}_{m-1}}{c} \\ \\ (\hat{p}_{zz})_{m-1} \\ \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{\dot{\hat{u}}_{m-1}}{c} \\ \\ \dfrac{\dot{\hat{w}}_{m}}{c} \\ \\ (\hat{p}_{zz})_{m} \\ \\ 0 \end{bmatrix}$$

where the fourth element of the column vector is the tangential stress which is zero at a solid-liquid boundary. The matrix elements are given in Appendix A and may be derived following Haskell (1953). Equation (3) yields the three equations

$$\frac{\dot{\hat{u}}_{m-1}}{c} = -\frac{(a_m)_{42}}{(a_m)_{41}}\frac{\dot{\hat{w}}_{m-1}}{c} - \frac{(a_m)_{43}}{(a_m)_{41}}(\hat{p}_{zz})_{m-1} \tag{4}$$

$$\frac{\dot{\hat{w}}_m}{c} = (a_m)_{21}\frac{\dot{\hat{u}}_{m-1}}{c} + (a_m)_{22}\frac{\dot{\hat{w}}_{m-1}}{c} + (a_m)_{23}(\hat{p}_{zz})_{m-1} \tag{5}$$

$$(\hat{p}_{zz})_m = (a_m)_{31}\frac{\dot{\hat{u}}_{m-1}}{c} + (a_m)_{32}\frac{\dot{\hat{w}}_{m-1}}{c} + (a_m)_{33}(\hat{p}_{zz})_{m-1} \,. \tag{6}$$

Substituting $\frac{\dot{\hat{u}}_{m-1}}{c}$ from Equation (4) in Equations (5) and (6) yields

$$\frac{\dot{\hat{w}}_m}{c} = \left[(a_m)_{22} - \frac{(a_m)_{21}(a_m)_{42}}{(a_m)_{41}}\right]\frac{\dot{\hat{w}}_{m-1}}{c} + \left[(a_m)_{23} - \frac{(a_m)_{21}(a_m)_{43}}{(a_m)_{41}}\right](\hat{p}_{zz})_{m-1} \tag{7}$$

$$(\hat{p}_{zz})_m = \left[(a_m)_{32} - \frac{(a_m)_{31}(a_m)_{42}}{(a_m)_{41}}\right]\frac{\dot{\hat{w}}_{m-1}}{c} + \left[(a_m)_{33} - \frac{(a_m)_{31}(a_m)_{43}}{(a_m)_{41}}\right](\hat{p}_{zz})_{m-1}$$

or in matrix notation

$$\begin{bmatrix} \dfrac{\dot{\hat{w}}_m}{c} \\ (\hat{p}_{zz})_m \end{bmatrix} = \begin{bmatrix} (a_m)_{22} - \dfrac{(a_m)_{21}(a_m)_{42}}{(a_m)_{41}} & (a_m)_{23} - \dfrac{(a_m)_{21}(a_m)_{43}}{(a_m)_{41}} \\ (a_m)_{32} - \dfrac{(a_m)_{31}(a_m)_{42}}{(a_m)_{41}} & (a_m)_{33} - \dfrac{(a_m)_{31}(a_m)_{43}}{(a_m)_{41}} \end{bmatrix}$$

$$\cdot \begin{bmatrix} \dfrac{\dot{\hat{w}}_{m-1}}{c} \\ (\hat{p}_{zz})_{m-1} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \dfrac{\dot{\hat{w}}_{m-1}}{c} \\ (\hat{p}_{zz})_{m-1} \end{bmatrix}$$

which is the same as the matrix relation for a liquid layer. In general, then, for interbedded solids and liquids

$$\begin{bmatrix} \dfrac{\dot{\hat{w}}_{n-1}}{c} \\ (\hat{p}_{zz})_{n-1} \end{bmatrix} = a_{n-1} \cdots b_{n-3} a_{n-4} \cdots b_1 \begin{bmatrix} \dfrac{\dot{\hat{w}}_0}{c} \\ 0 \end{bmatrix}$$

If layers ℓ to $\ell + k$ are solid layers in sequence we have

$$A_{\ell+k} = a_{\ell+k} \cdots a_k$$

where each layer matrix is for a solid layer. The product matrix for the solid layers between liquid layers may then be written

$$B_{\ell+k} = \begin{bmatrix} (B_{\ell+k})_{11} & (B_{\ell+k})_{12} \\ (B_{\ell+k})_{21} & (B_{\ell+k})_{22} \end{bmatrix}$$

where

$$(B_{\ell+k})_{11} = (A_{\ell+k})_{22} - \frac{(A_{\ell+k})_{21}(A_{\ell+k})_{42}}{(A_{\ell+k})_{41}}$$

$$(B_{\ell+k})_{12} = (A_{\ell+k})_{23} - \frac{(A_{\ell+k})_{21}(A_{\ell+k})_{43}}{(A_{\ell+k})_{41}}$$

$$(B_{\ell+k})_{21} = (A_{\ell+k})_{32} - \frac{(A_{\ell+k})_{31}(A_{\ell+k})_{42}}{(A_{\ell+k})_{41}}$$

$$(B_{\ell+k})_{22} = (A_{\ell+k})_{33} - \frac{(A_{\ell+k})_{31}(A_{\ell+k})_{43}}{(A_{\ell+k})_{41}} .$$

Point Source of Harmonic Waves in a Liquid Layer

We divide the liquid source layer into two layers as shown in Figure 1. At $z = D'$ the pressure is continuous. The vertical particle velocity is continuous everywhere in the plane defined by $z = D'$ except at the point source where the liquid above and below the source moves in opposite directions. This may be expressed by writing

$$\delta\left(\frac{\hat{\omega}'_s}{c}\right) = \frac{2k}{c}$$

For the liquid source layer

$$\begin{bmatrix} \dfrac{\dot{\hat{u}}_{q-1}}{c} \\ \dfrac{\dot{\hat{w}}_{s_2}(D)}{c} \\ (\hat{p}_{zz})_{s_2}(D) \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{\dot{\hat{u}}_{q-1}}{c} \\ \dfrac{\dot{\hat{w}}_{s_1}(D)}{c} \\ (\hat{p}_{zz})_{s_1}(D) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \delta\left(\dfrac{\dot{\hat{w}}_s}{c}\right) \\ 0 \\ 0 \end{bmatrix} \qquad (8)$$

where $\dot{\hat{u}}_{q-1}$ is the horizontal particle velocity at the bottom of the first liquid layer above the half space. For the layers below $z = D'$

$$\begin{bmatrix} \dfrac{\dot{\hat{u}}_{n-1}}{c} \\ \dfrac{\dot{\hat{w}}_{n-1}}{c} \\ (\hat{p}_{zz})_{n-1} \\ \hat{\tau}_{n-1} \end{bmatrix} = A_{s_2} \begin{bmatrix} \dfrac{\dot{\hat{u}}_{q-1}}{c} \\ \dfrac{\dot{\hat{w}}_{s_2}(D)}{c} \\ (\hat{p}_{zz})_{s_2}(D) \\ 0 \end{bmatrix} \qquad (9)$$

and for the layers above z = D'

$$\begin{bmatrix} \dfrac{\hat{\acute{u}}_{q-1}}{c} \\ \dfrac{\hat{\acute{w}}_{s_1}(D)}{c} \\ (\hat{p}_{zz})_{s_1}(D) \\ 0 \end{bmatrix} = A_{s_1} \begin{bmatrix} \dfrac{\hat{\acute{u}}_{q-1}}{c} \\ \dfrac{\hat{\acute{w}}_0}{c} \\ 0 \\ 0 \end{bmatrix} . \tag{10}$$

At the free surface both $(\hat{p}_{zz})_0$ and $\hat{\tau}_0$ are zero.

In Equations 9 and 10

$$A_{s_2} = a_{n-1} \cdots a_{n-5}\, b_{n-6} \cdots a_{s_2}$$

$$A_{s_1} = a_{s_1}\, b_{s_1-1} \cdots a_2\, b_1 \; .$$

For the liquid layers in terms of 4 x 4 matrices

$$a_m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (a_m)_{22} & (a_m)_{23} & 0 \\ 0 & (a_m)_{32} & (a_m)_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and for solid layers between liquid layers in terms of 4 by 4 matrices

$$b_m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (b_m)_{22} & (b_m)_{23} & 0 \\ 0 & (b_m)_{32} & (b_m)_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $(b_m)_{22}$, $(b_m)_{23}$, $(b_m)_{32}$, $(b_m)_{33}$ are given by Equations 7 in terms of elements of the solid layer matrix

$$a_m = \begin{bmatrix} (a_m)_{11} & (a_m)_{12} & (a_m)_{13} & (a_m)_{14} \\ (a_m)_{21} & (a_m)_{22} & (a_m)_{23} & (a_m)_{24} \\ (a_m)_{31} & (a_m)_{32} & (a_m)_{33} & (a_m)_{34} \\ (a_m)_{41} & (a_m)_{42} & (a_m)_{43} & (a_m)_{44} \end{bmatrix}$$

We now write for the solid half-space (Haskell, 1953)

$$\begin{bmatrix} \Delta_n' + \Delta_n'' \\ \Delta_n' - \Delta_n'' \\ \omega_n' + \omega_n'' \\ \omega_n' - \omega_n'' \end{bmatrix} = E^{-1} \begin{bmatrix} \dfrac{\dot{\hat{u}}_{n-1}}{c} \\ \dfrac{\dot{\hat{w}}_{n-1}}{c} \\ (\hat{p}_{zz})_{n-1} \\ (\hat{\tau})_{n-1} \end{bmatrix}$$

where

$$E^{-1} = \begin{bmatrix} -2\left(\frac{\beta_n}{\alpha_n}\right)^2 & 0 & (\rho_n \alpha_n^2)^{-1} & 0 \\ 0 & \frac{c^2(\gamma_n - 1)}{\alpha_n^2 r_{\alpha_n}} & 0 & (\rho_n \alpha_n^2 r_{\alpha_n})^{-1} \\ \frac{\gamma_n - 1}{\gamma_n r_{\beta_n}} & 0 & -(\rho_n c^2 \gamma_n r_{\beta_n})^{-1} & 0 \\ 0 & 1 & 0 & (\rho_n c^2 \gamma_n)^{-1} \end{bmatrix}$$

and Δ_n', Δ_n'', ω_n', ω_n'' are constants.

Applying the condition that no sources radiate from infinity we set Δ_n'', ω_n'' equal to zero and hence

$$\begin{bmatrix} \Delta_n' \\ \Delta_n' \\ \omega_n' \\ \omega_n' \end{bmatrix} = E^{-1} \begin{bmatrix} \frac{\hat{u}_{n-1}}{c} \\ \frac{\hat{w}_{n-1}}{c} \\ (\hat{p}_{zz})_{n-1} \\ (\hat{\tau})_{n-1} \end{bmatrix} . \tag{11}$$

From Equation 9, Equation 11 may be written

$$\begin{bmatrix} \Delta_n' \\ \Delta_n' \\ \omega_n' \\ \omega_n' \end{bmatrix} = E^{-1} A_{s_2} \begin{bmatrix} \frac{\hat{u}_{q-1}}{c} \\ \frac{\hat{w}_{s_2}(D)}{c} \\ (\hat{p}_{zz})_{s_2}(D) \\ 0 \end{bmatrix}$$

or from Equation 8

$$\begin{bmatrix} \Delta_n' \\ \Delta_n' \\ \omega_n' \\ \omega_n' \end{bmatrix} = E^{-1} A_{s_2} \left\{ \begin{bmatrix} \frac{\hat{u}'_{g-1}}{c} \\ \frac{\hat{w}'_{s_1}(D)}{c} \\ (p_{zz})_{s_1}(D) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \delta\left(\frac{\hat{w}'_s}{c}\right) \\ 0 \\ 0 \end{bmatrix} \right\}$$

and from Equation 10

$$\begin{bmatrix} \Delta_n' \\ \Delta_n' \\ \omega_n' \\ \omega_n' \end{bmatrix} = E^{-1} \left\{ A_{s_2} A_{s_1} \begin{bmatrix} \frac{\hat{u}'_{g-1}}{c} \\ \frac{\hat{w}'_0}{c} \\ 0 \\ 0 \end{bmatrix} + A_{s_2} \begin{bmatrix} 0 \\ \delta\left(\frac{\hat{w}'_s}{c}\right) \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$= E^{-1} A \left\{ \begin{bmatrix} \frac{\hat{u}'_{g-1}}{c} \\ \frac{\hat{w}'_0}{c} \\ 0 \\ 0 \end{bmatrix} + A_{s_1}^{-1} \begin{bmatrix} 0 \\ \delta\left(\frac{\hat{w}'_s}{c}\right) \\ 0 \\ 0 \end{bmatrix} \right\} \qquad (12)$$

where

$$A = A_{s_2} A_{s_1} .$$

Let

$$\begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \frac{\hat{\dot{u}}_{q-1}}{c} \\ \frac{\hat{\dot{w}}_0}{c} \\ 0 \\ 0 \end{bmatrix} + A_{s_1}^{-1} \begin{bmatrix} 0 \\ \delta\left(\frac{\hat{\dot{w}}_s}{c}\right) \\ 0 \\ 0 \end{bmatrix} .$$

We may prove following Harkrider (1964) that the inverse of the product matrix A_{s_1} has the same form as the inverse of a layer matrix and hence

$$A_{s_1}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (A_{s_1})_{33} & -(A_{s_1})_{23} & 0 \\ 0 & -(A_{s_1})_{32} & (A_{s_1})_{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

Therefore

$$\begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \frac{\hat{\dot{u}}_{q-1}}{c} \\ \frac{\hat{\dot{w}}_0}{c} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (A_s)_{33} & -(A_{s_1})_{23} & 0 \\ 0 & -(A_{s_1})_{32} & (A_{s_1})_{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \delta\left(\frac{\hat{\dot{w}}_s}{c}\right) \\ 0 \\ 0 \end{bmatrix}$$

or in terms of matrix elements

$$W = \frac{\hat{u}_{q-1}}{c} \tag{13}$$

$$X = \frac{\hat{\dot{w}}_0}{c} + (A_{s_1})_{33}\,\delta\!\left(\frac{\hat{\dot{w}}_s}{c}\right) \tag{14}$$

(15)

$$Y = -(A_{s_1})_{32}\,\delta\!\left(\frac{\hat{\dot{w}}_s}{c}\right) \tag{16}$$

$$Z = 0 \quad .$$

We define $J = E^{-1}A$ and write Equation (12)

$$\begin{bmatrix} \Delta_n' \\ \Delta_n' \\ \omega_n' \\ \omega_n' \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{bmatrix} \begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} .$$

Eliminating Δ_n'

$$(J_{11} - J_{21})W + (J_{12} - J_{22})X + (J_{13} - J_{23})Y + (J_{14} - J_{24})Z = 0 \tag{17}$$

and eliminating ω_n'

$$(J_{31} - J_{41})W + (J_{32} - J_{42})X + (J_{33} - J_{43})Y + (J_{34} - J_{44})Z = 0 \quad . \tag{18}$$

If we let

$$L = J_{11} - J_{21}$$
$$K = J_{12} - J_{22}$$
$$G = J_{13} - J_{23}$$
$$R = J_{14} - J_{24}$$
$$N = J_{31} - J_{41}$$
$$M = J_{32} - J_{42}$$
$$H = J_{33} - J_{43}$$
$$S = J_{34} - J_{44}$$

and solve Equations (17) and (18) for W and X respectively,

$$W = -\frac{K}{L}X - \frac{G}{L}Y - \frac{R}{L}Z \tag{19}$$

$$X = -\frac{N}{M}W - \frac{H}{M}Y - \frac{S}{M}Z \tag{20}$$

or substituting Equation 19 in Equation 20

$$X = -\frac{N}{M}\left(-\frac{K}{L}X - \frac{G}{L}Y - \frac{R}{L}Z\right) - \frac{H}{M}Y - \frac{S}{M}Z$$

and solving for X

$$X = \frac{(GN-HL)Y + (RN-SL)Z}{ML-NK}$$

and therefore from Equations (15) and (16)

$$X = \frac{(GN-HL)(A_{s_1})_{32}}{NK-ML}\,\delta\left(\frac{\hat{\omega}'_s}{c}\right)$$

or from the Equation (14)

$$\frac{\hat{\omega}'_o}{c} = \frac{(GN-HL)(A_{s_1})_{32}}{NK-ML}\,\delta\left(\frac{\hat{\omega}'_s}{c}\right) - (A_{s_1})_{33}\,\delta\left(\frac{\hat{\omega}'_s}{c}\right)$$

Hence, the integral solution is

$$\frac{\dot{w}_0}{c} = \int_0^\infty e^{i\omega t}\left[\frac{(GN-HL)(A_{s_1})_{32}-(NK-ML)(A_{s_1})_{33}}{NK-ML}\right] \cdot \frac{2k}{c} J_0(kr)\,dk \quad .$$

The integral solution for the horizontal particle velocity at the surface of an ice sheet is by Equation 4

$$\frac{\dot{u}_0}{c} = -\int_0^\infty e^{i\omega t}\frac{(a_1)_{42}}{(a_1)_{41}}\frac{\hat{\dot{w}}_0}{iC} J_1(kr)\,dk$$

where

$$-\frac{(a_1)_{42}}{(a_1)_{41}} = \frac{\hat{\dot{u}}_0}{\hat{\dot{w}}_0} = \frac{\hat{u}_0}{\hat{w}_0}$$

is the ratio of horizontal to vertical particle velocity or particle displacement at the surface.

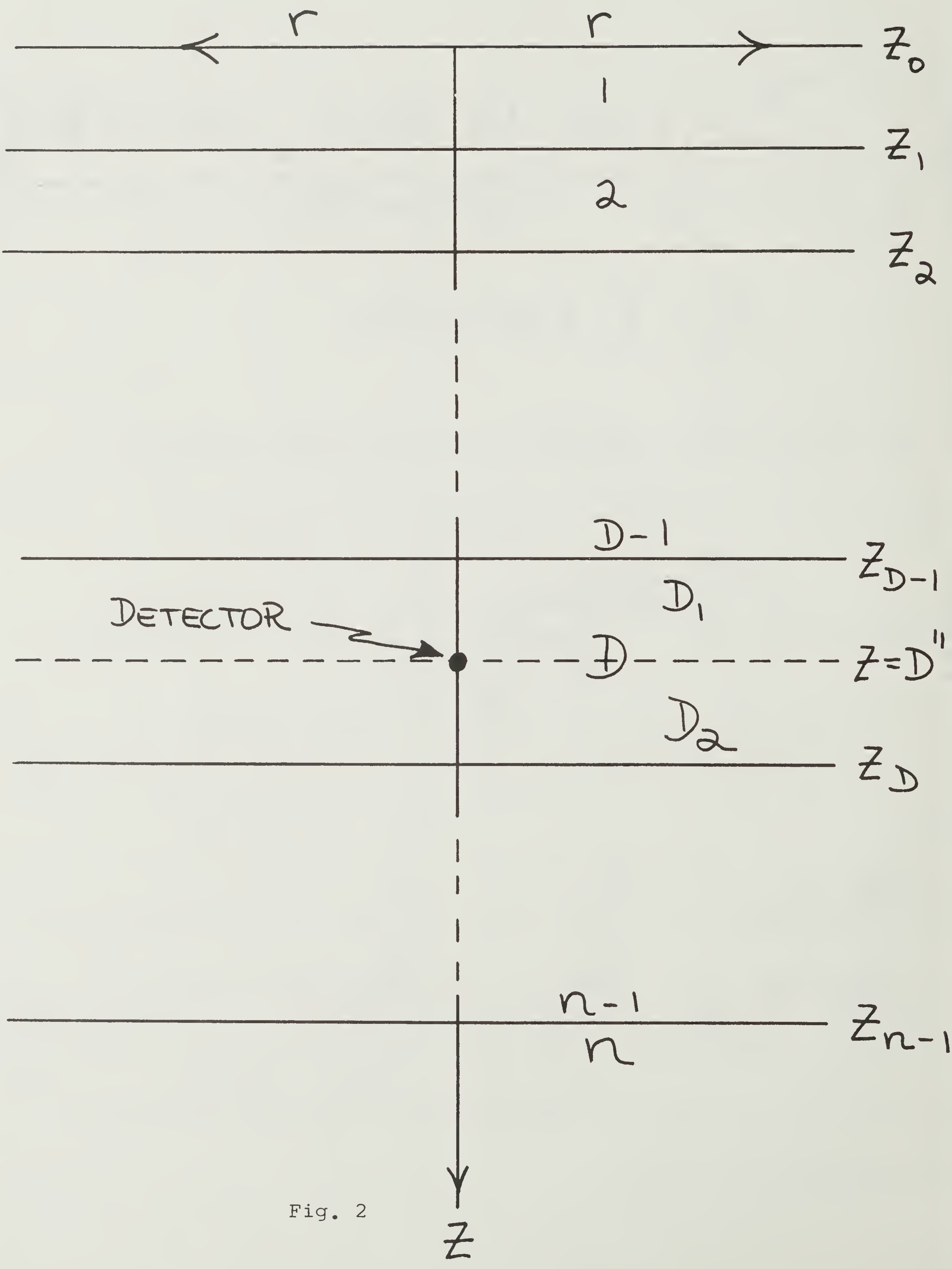


Fig. 2

If a hydrophone or vertical particle velocity detector is located in a liquid layer at the bottom of the D_1 layer in Figure 2,

$$\begin{bmatrix} \frac{\dot{\hat{u}}_{q-1}}{c} \\ \frac{\dot{\hat{w}}_{D_1}}{c} \\ (\hat{p}_{zz})_{D_1} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (A_{D_1})_{22} & (A_{D_1})_{23} & 0 \\ 0 & (A_{D_1})_{32} & (A_{D_1})_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\dot{\hat{u}}_{q-1}}{c} \\ \frac{\dot{\hat{w}}_0}{c} \\ 0 \\ 0 \end{bmatrix}$$

where

$$A_{D_1} = a_{D_1} b_{D_1 - 1} \cdots a_2 b_1$$

or

$$\frac{\dot{\hat{w}}_{D_1}}{c} = (A_{D_1})_{22} \frac{\dot{\hat{w}}_0}{c}$$

$$(p_{zz})_{D_1} = -p_{D_1} = (A_{D_1})_{32} \frac{\dot{\hat{w}}_0}{c} .$$

Hence, the integral solution for pressure at the bottom of layer D_1 is

$$p_{D_1} = -\int_0^\infty e^{i\omega t} \left[\frac{(GN-HL)(A_{S_1})_{32} - (NK-ML)(A_{S_1})_{33}}{NK-ML} \right] \cdot (A_{D_1})_{32} \frac{2k}{c} J_0(kr)\, dk$$

and the vertical particle velocity at the bottom of the D_1 layer is

$$\frac{\dot{w}_{D_1}}{c} = \int_0^{\infty} e^{i\omega t}\left[\frac{(GN-HL)(A_{S_1})_{32}-(NK-ML)(A_{S_1})_{33}}{NK-ML}\right] \cdot \frac{2k}{c}(A_{D_1})_{22}\, J_0(kr)\,dk \quad .$$

In particular at the ocean bottom

$$\frac{\dot{w}_{D_1}}{c} = \frac{\dot{w}_{q-1}}{c} \quad .$$

The corresponding horizontal particle velocity at the ocean bottom is obtained from the expression

$$\begin{bmatrix} \Delta_n' \\ \Delta_n' \\ w_n' \\ w_n' \end{bmatrix} = J \begin{bmatrix} \frac{\hat{\dot{u}}_{q-1}}{c} \\ \frac{\hat{\dot{w}}_0}{c} \\ 0 \\ 0 \end{bmatrix}$$

or by eliminating Δ_n'

$$\frac{\hat{\dot{u}}_{q-1}}{\hat{\dot{w}}_0} = -\frac{K}{L} \quad .$$

Therefore

$$\frac{\dot{u}_{q-1}}{c} = -\int_0^{\infty} e^{i\omega t}\,\frac{K}{L}\left[\frac{(GN-HL)(A_{s_1})_{32} - (NK-ML)(A_{s_1})_{33}}{NK-ML}\right]$$

$$\cdot \frac{2k}{ic}\,J_1(kr)\,dk \;.$$

Carrying out the matrix multiplication

$$\begin{bmatrix} \Delta_n' \\ \Delta_n' \\ \omega_n' \\ \omega_n' \end{bmatrix} = E^{-1}A \begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix}$$

and eliminating Δ_n' and ω_n' explicit expressions for

L, K, G, N, M and H are

$$L = \gamma_n A_{11} + \frac{\gamma_n - 1}{r_{\alpha_n}} A_{21} - \frac{A_{31}}{\rho_n c^2} + \frac{A_{41}}{\rho_n c^2 r_{\alpha_n}}$$

$$K = \gamma_n A_{12} + \frac{\gamma_n - 1}{r_{\alpha_n}} A_{22} - \frac{A_{32}}{\rho_n c^2} + \frac{A_{42}}{\rho_n c^2 r_{\alpha_n}}$$

$$G = \gamma_n A_{13} + \frac{\gamma_n - 1}{r_{\alpha_n}} A_{23} - \frac{A_{33}}{\rho_n c^2} + \frac{A_{43}}{\rho_n c^2 r_{\alpha_n}}$$

$$N = -\frac{\gamma_n - 1}{r_{\beta_n}} A_{11} + \gamma_n A_{21} + \frac{A_{31}}{\rho_n c^2 r_{\beta_n}} + \frac{A_{41}}{\rho_n c^2}$$

$$M = -\frac{\gamma_n - 1}{r_{\beta_n}} A_{12} + \gamma_n A_{22} + \frac{A_{32}}{\rho_n c^2 r_{\beta_n}} + \frac{A_{42}}{\rho_n c^2}$$

$$H = -\frac{\gamma_n - 1}{r_{\beta_n}} A_{13} + \gamma_n A_{23} + \frac{A_{33}}{\rho_n c^2 r_{\beta_n}} + \frac{A_{43}}{\rho_n c^2}$$

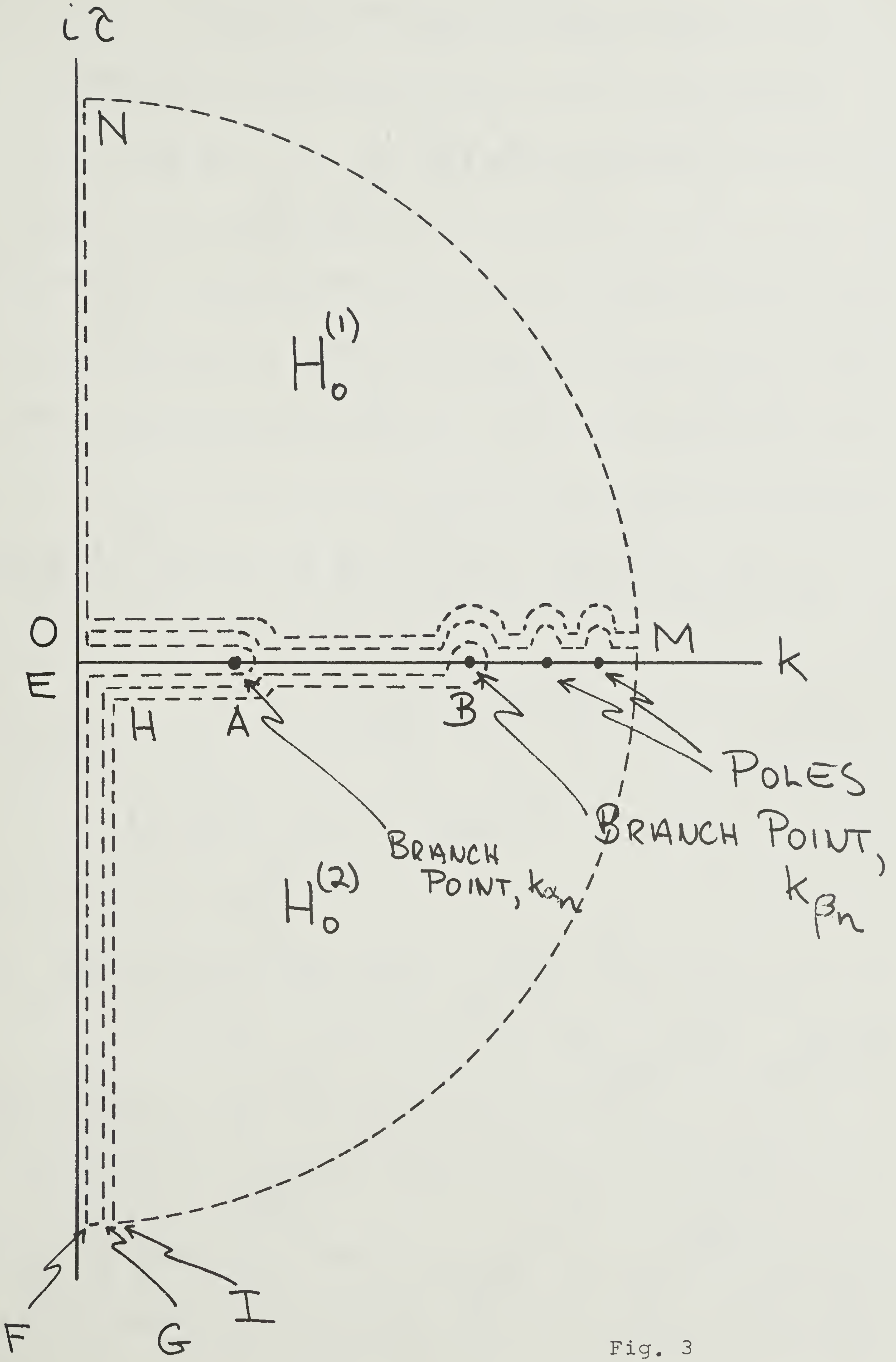

$i\tau$
N
$H_0^{(1)}$
O
M
K
E
H
A
B
POLES
BRANCH POINT, k_{β_n}
BRANCH POINT, k_{α_n}
$H_0^{(2)}$
F
G
I

Fig. 3

EVALUATION OF THE INTEGRAL SOLUTIONS

The integral solutions have singularities corresponding to

$$(NK - ML)_{k=k_e} = 0$$

These are simple poles for real wave number and the integral solutions are conveniently evaluated by contour integration in the complex $\xi = k + i\tau$ plane. The contours are shown in Figure 3. First the transformation

$$2 J_0(\xi r) = H_0^{(1)}(\xi r) + H_0^{(2)}(\xi r)$$

is made. In addition to simple poles at

$$(NK - ML)_{k=k_e} = 0$$

branch points occur at $k r_{\alpha_m}$ for the liquid layers and $k r_{\alpha_p}, k r_{\beta_p}$ for each solid layer.

To make the integrand single valued we must introduce branch cuts originating at the branch points $k r_{\alpha_m}$ for each liquid layer, and $k r_{\alpha_p}, k r_{\beta_p}$ for each solid layer. We may then apply Cauchy's Integral Theorem. We note that since the integrands are

even functions of $k r_{\alpha_m}$, $(k r_{\alpha_p}, k r_{\beta_p})$ branch line integrals corresponding to branch cuts made to these branch points cancel and only the branch cuts for $k r_{\alpha_n}, k r_{\beta_n}$ must be considered.

The Riemann surface has four sheets. To satisfy the vanishing of $\dot{w}_0$ as ζ approaches infinity we must remain on the sheet where the real part of $i\zeta r_{\alpha_n}$ and $i\zeta r_{\beta_n}$ is greater than zero. For the contours shown in Figure 3 complex poles have been displaced to an unused Riemann surface (Ewing et al 1957, pp 136-137).

The integral for $\dot{w}_0$, dropping the $e^{i\omega t}$ term, is now transformed in the upper half-plane to

$$\int_0^M I_1\,d\zeta + \int_{C(M,N)} I_1\,d\zeta + \int_N^O I_1\,d\zeta = 0 \qquad (21)$$

and in the lower half-plane to

$$\begin{aligned}&\int_E^F I_2\,d\zeta + \int_G^H I_2\,d\zeta + \int_H^A I_2\,d\zeta + \\ &\int_A^B I_2\,d\zeta + \int_B^A I_2\,d\zeta + \int_A^H I_2\,d\zeta + \\ &\int_H^I I_2\,d\zeta + \int_{C(I,M)} I_2\,d\zeta + \int_M^O I_2\,d\zeta +\end{aligned} \qquad (22)$$

$$\int_{OAE} I_2 \, d\zeta = 2\pi i \sum \mathrm{Res}(I_2)$$

where

$$I_1 = \frac{k}{c}\left[\frac{(GN-HL)(A_{s_1})_{32} - (NK-ML)(A_{s_1})_{33}}{NK-ML}\right] H_0^{(1)}(\zeta r)$$

$$I_2 = \frac{k}{c}\left[\frac{(GN-HL)(A_{s_1})_{32} - (NK-ML)(A_{s_1})_{33}}{NK-ML}\right] H_0^{(2)}(\zeta r) .$$

Adding Equations 21 and 22 and noting that the integrals along the infinite arcs such as $C(M,N)$ are zero we have the desired result

$$\dot{w}_0 = \int_0^\infty I_1 \, dk + \int_0^\infty I_2 \, dk = \sum \int_{\text{BRANCH CUTS}}$$

$$- 2\pi i \sum \mathrm{Res}(I_2) .$$

The branch line integrals are difficult to evaluate numerically and discussion of them is omitted, but we note that the proper signs for the radicals r_{α_n}, r_{β_n} must be made on each side of a cut.

We have as our final result for the normal modes for $\dot{w}_0$ and

$$\dot{w}_0 = -\pi i \sum_{l} \left\{ \frac{(GN-HL)(A_{s_1})_{32}\, 2k\, H_0^{(2)}(kr)}{\frac{\partial}{\partial k}(NK-ML)} \right\}_{k=k_l}$$

$$p_{D_1} = \pi i \sum_{l} \left\{ \frac{(GN-HL)(A_{s_1})_{32}(A_{D_1})_{32}\, 2k\, H_0^{(2)}(kr)}{c\,\frac{\partial}{\partial k}(NK-ML)} \right\}_{k=k_l}$$

where the period equation (phase velocity dispersion) $(KN-LM)_{k=k_l}=0$ has been used to simplify the expressions. Group velocity dispersion is given by $U=\frac{d\omega}{dk}$.

Corresponding expressions may be written for the vertical particle velocity at depth or the horizontal particle velocity at the ice surface or on the ocean bottom.

For a constant pressure source we multiply $\dot{w}_0$ and p_{D_1} by $\frac{P_0}{i\omega\rho_s}$, where P_0 is the pressure at unit distance from the source, and write

$$\dot{w}_0 = \pi P_0 \sum_{l} \left\{ \frac{(GN-HL)(A_{s_1})_{32}\, 2\omega\, H_0^{(2)}(kr)}{c^3 \rho_s \frac{\partial}{\partial c}(NK-ML)} \right\}_{c=c_l}$$

$$p_{D_1} = -\pi P_0 \sum_{\ell} \left\{ \frac{(GN-HL)(A_{S_1})_{32}(A_{D_1})_{32} 2\omega H_0^{(2)}(kr)}{c^4 \rho_s \frac{\partial}{\partial c}(NK-Mh)} \right\}_{c=c_\ell} .$$

Let

$$\Delta = ML - Im(N)\,Im(-K)$$

$$\bar{\Delta} = L\,Im(H) - G\,Im(N) .$$

For programming it is convenient to write the expressions for vertical particle velocity at the surface and pressure in the form

$$\dot{w}_0 = 2\pi P_0 \sum_{\ell} \left\{ \frac{\omega \bar{\Delta}\, Im\left[-(A_{S_1})_{32}\right] H_0^{(2)}(kr)}{c^3 \rho_s \frac{\partial \Delta}{\partial c}} \right\}_{c=c_\ell} \qquad (23)$$

$$p_{D_1} = 2\pi i P_0 \sum_{\ell} \left\{ \frac{\omega \bar{\Delta}\, Im\left[-(A_{S_1})_{32}\right] Im\left[-(A_{D_1})_{32}\right] H_0^{(2)}(kr)}{c^4 \rho_s \frac{\partial \Delta}{\partial c}} \right\}_{c=c_\ell} . \qquad (24)$$

If we set

$$H_0^{(2)}(kr) = J_0(kr) - iY_0(kr)$$

in Equations 23 and 24, we have

$$\dot{w}_0 = W_1 - iW_2$$

and for the absolute value of the vertical particle velocity, a quantity conveniently measured in transmission experiments

$$|\dot{w}_0| = \sqrt{W_1^2 + W_2^2}$$

where

$$W_1 = 2\pi P_0 \sum_{\ell} \left\{ \frac{\omega \bar{\Delta}\, Im\left[-(A_{s_1})_{32}\right] J_0(kr)}{c^3 \rho_s \frac{\partial \Delta}{\partial c}} \right\}_{c=c_\ell}$$

$$W_2 = 2\pi P_0 \sum_{\ell} \left\{ \frac{\omega \bar{\Delta}\, \mathrm{Im}\left[-(A_{s,1})_{32}\right] Y_0(kr)}{c^3 \rho_s \frac{\partial \Delta}{\partial c}} \right\}_{c=c_\ell} .$$

Likewise for the vertical particle displacement, multiplying

$\dot{w}_0$ by $\frac{1}{i\omega}$

we have

$$w_0 = -W_3 - iW_4$$

or

$$|w_0| = \sqrt{W_3^2 + W_4^2}$$

where

$$W_3 = 2\pi P_0 \sum_{\ell} \left\{ \frac{\bar{\Delta}\, \mathrm{Im}\left[-(A_{s,1})_{32}\right] Y_0(kr)}{c^3 \rho_s \frac{\partial \Delta}{\partial c}} \right\}_{c=c_\ell}$$

$$W_4 = 2\pi P_0 \sum_{\ell} \left\{ \frac{\bar{\Delta}\, \mathrm{Im}\left[-(A_{s,1})_{32}\right] J_0(kr)}{c^3 \rho_s \frac{\partial \Delta}{\partial c}} \right\}_{c=c_\ell}$$

and for pressure

$$p_{D_1} = W_5 + i W_6 \quad ,$$

$$|p_{D_1}| = \sqrt{W_5^2 + W_6^2}$$

where

$$W_5 = 2\pi P_0 \sum_{\ell} \left\{ \frac{\omega \bar{\Delta}\, Im\left[-(A_{S_1})_{32}\right] Im\left[-(A_{D_1})_{32}\right] Y_0(kr)}{c^4 \rho_S \dfrac{\partial \Delta}{\partial c}} \right\}_{c=c_\ell}$$

$$W_6 = 2\pi P_0 \sum_{\ell} \left\{ \frac{\omega \bar{\Delta}\, Im\left[-(A_{S_1})_{32}\right] Im\left[-(A_{D_1})_{32}\right] J_0(kr)}{c^4 \rho_S \dfrac{\partial \Delta}{\partial c}} \right\}_{c=c_\ell}$$

The solid layer matrices are programmed as

$$\begin{bmatrix} (a_m)_{11} & -Im\left[(a_m)_{12}\right] & (a_m)_{13} & -Im\left[(a_m)_{14}\right] \\ Im\left[(a_m)_{21}\right] & (a_m)_{22} & Im\left[(a_m)_{23}\right] & (a_m)_{24} \\ (a_m)_{31} & -Im\left[(a_m)_{32}\right] & (a_m)_{33} & -Im\left[(a_m)_{34}\right] \\ Im\left[(a_m)_{41}\right] & (a_m)_{42} & Im\left[(a_m)_{43}\right] & (a_m)_{44} \end{bmatrix}$$

or for solid layers between liquid layers

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (b_m)_{22} & \mathrm{Im}[(b_m)_{23}] & 0 \\ 0 & -\mathrm{Im}[(b_m)_{32}] & (b_m)_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the liquid layer matrices as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (a_m)_{22} & \mathrm{Im}[(a_m)_{23}] & 0 \\ 0 & -\mathrm{Im}[(a_m)_{32}] & (a_m)_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

NUMERICAL COMPUTATIONS

The present formulas for the pressure in dynes/cm^2 and the vertical and horizontal particle displacements in millimicrons are incorporated in the same computer programs employed previously (Kutschale, 1970). Computations are made in two stages. The first program, an extension of Dorman's (1962) PV7 dispersion program, computes phase-and group-velocity dispersion, the variation of pressure with depth in the water, the ratio of horizontal to vertical particle motion in the ice and on the ocean bottom, the excitation function dependent only on the layering of the medium and the excitation function for the particular source and detector depths. For the l-th normal mode the excitation function of pressure dependent only on layering is defined by

$$\left(\frac{2\sqrt{2\pi\omega}\, P_0\, \bar{\Delta}}{C^{7/2}\, \dfrac{\partial \Delta}{\partial C}} \right)_{\substack{\omega = \text{CONSTANT} \\ C = C_\ell}}$$

and the excitation function for the particular source and detector depths is

$$\left\{ \frac{2\sqrt{2\pi\omega^{5}}\, P_0\, \bar{\Delta}}{C^{7/2}\, \dfrac{\partial \Delta}{\partial C}}\, \mathrm{Im}\left[-(A_{S_1})_{32}\right] \mathrm{Im}\left[-(A_{D_1})_{32}\right] \right\}_{\substack{\omega = \mathrm{const} \\ C = C_\ell}}$$

These definitions were chosen to be useful at long ranges where

$$H_0^{(2)}(kr) = \sqrt{\frac{2}{\pi k r}}\, e^{i(\pi/4 - kr)}$$

Corresponding expressions are defined for particle velocity and displacement. The second program computes and plots pressure or particle displacement as a function of range.

Sample numerical computations are presented for the layered Model A of Table 1. Layer thicknesses and compressional wave velocities are similar to those of Hunkins and Kutschale (1963) used for describing explosive sound transmission on the Chuckchi

TABLE 1

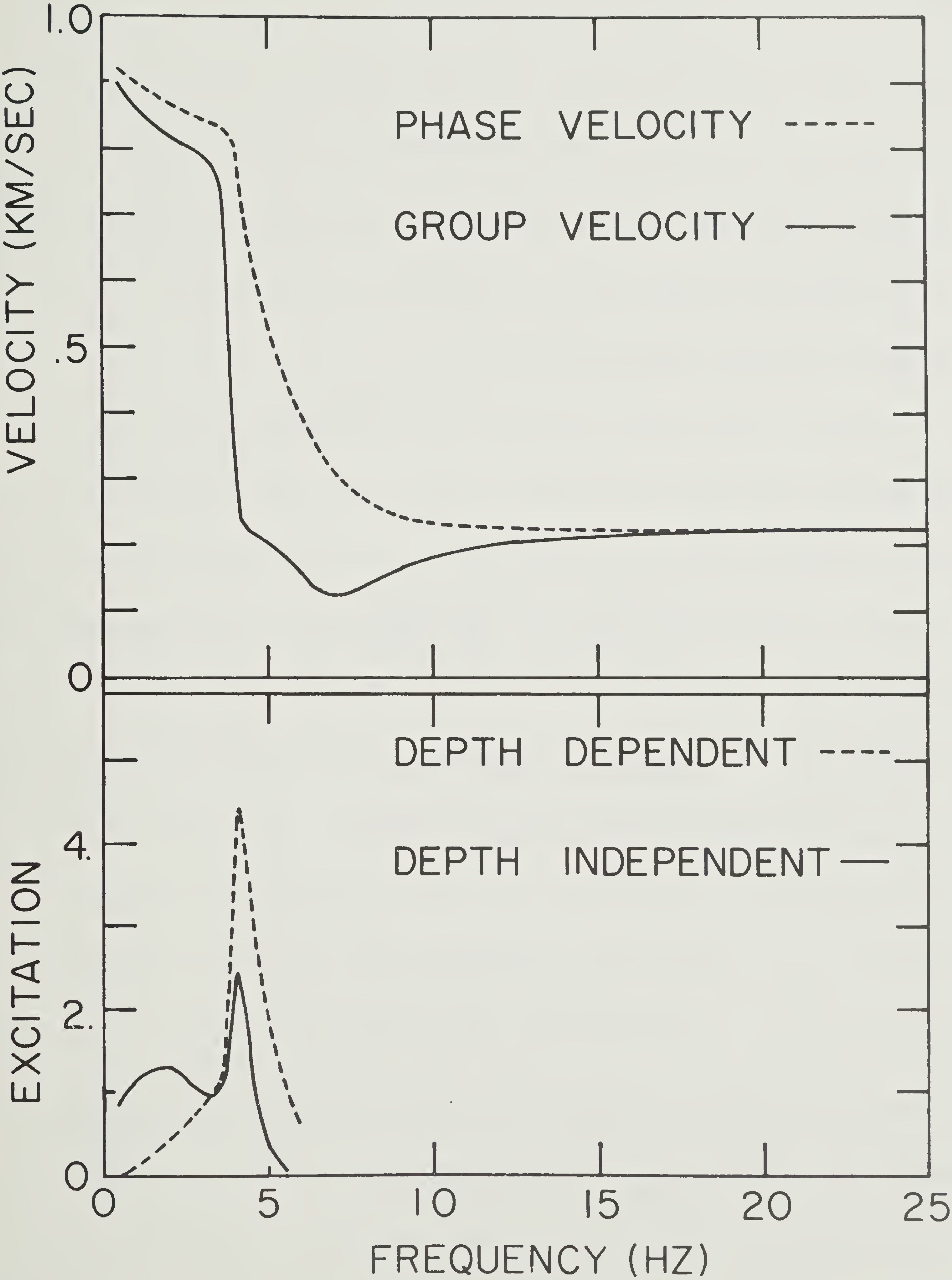
Layer Parameters for Model A

Layer	Thickness, m	Compressional-Wave Velocity, m/sec	Shear-Wave Velocity, m/sec	Density, gm/cm^3
Ice	3	3500.0	1800.0	0.900
Water	67	1440.0	0.0	1.025
Sediment	15	1750.0	250.0	1.800
Sediment	∞	2300.0	1000.0	2.050

TABLE 2

Source and Detector Parameters
for Model A

Source level at 1 m, dynes/cm^2	Source depth, m	Detector depth, m
100.0	50.0	50.0



1.0
.5
0
VELOCITY (KM/SEC)
PHASE VELOCITY
GROUP VELOCITY
DEPTH DEPENDENT
DEPTH INDEPENDENT
4.
2.
0
EXCITATION
0
5
10
15
20
25
FREQUENCY (HZ)

Fig. 4

Shelf north of Barrow, Alaska. The ice sheet is represented by a uniform layer 3 m thick. The assignment of shear-wave velocities to the sediments was made on the basis of observations of dispersion of Rayleigh waves from explosions detonated on the Chuckchi Shelf (Hunkins, personal communication).

Figure 4 shows phase and group velocity dispersion for the fundamental Rayleigh mode as well as the depth independent and depth dependent excitation functions. Source and detector depths for Model A are given in Table 2. The computations show that the ice sheet has very little effect on amplitudes in the water and on dispersion. The principal effect of the ice sheet is to change the polarization of the waves near the surface. If the ice sheet is neglected only vertical particle motion exists at the surface, but when the ice sheet is included in the layered system the waves are elliptically polarized at the surface and change their orbital motion through the ice sheet. This effect of the ice on particle motion near the surface is also observed for hydroacoustic waves traveling in the Arctic SOFAR channel (Kutschale, 1972). Figure 5

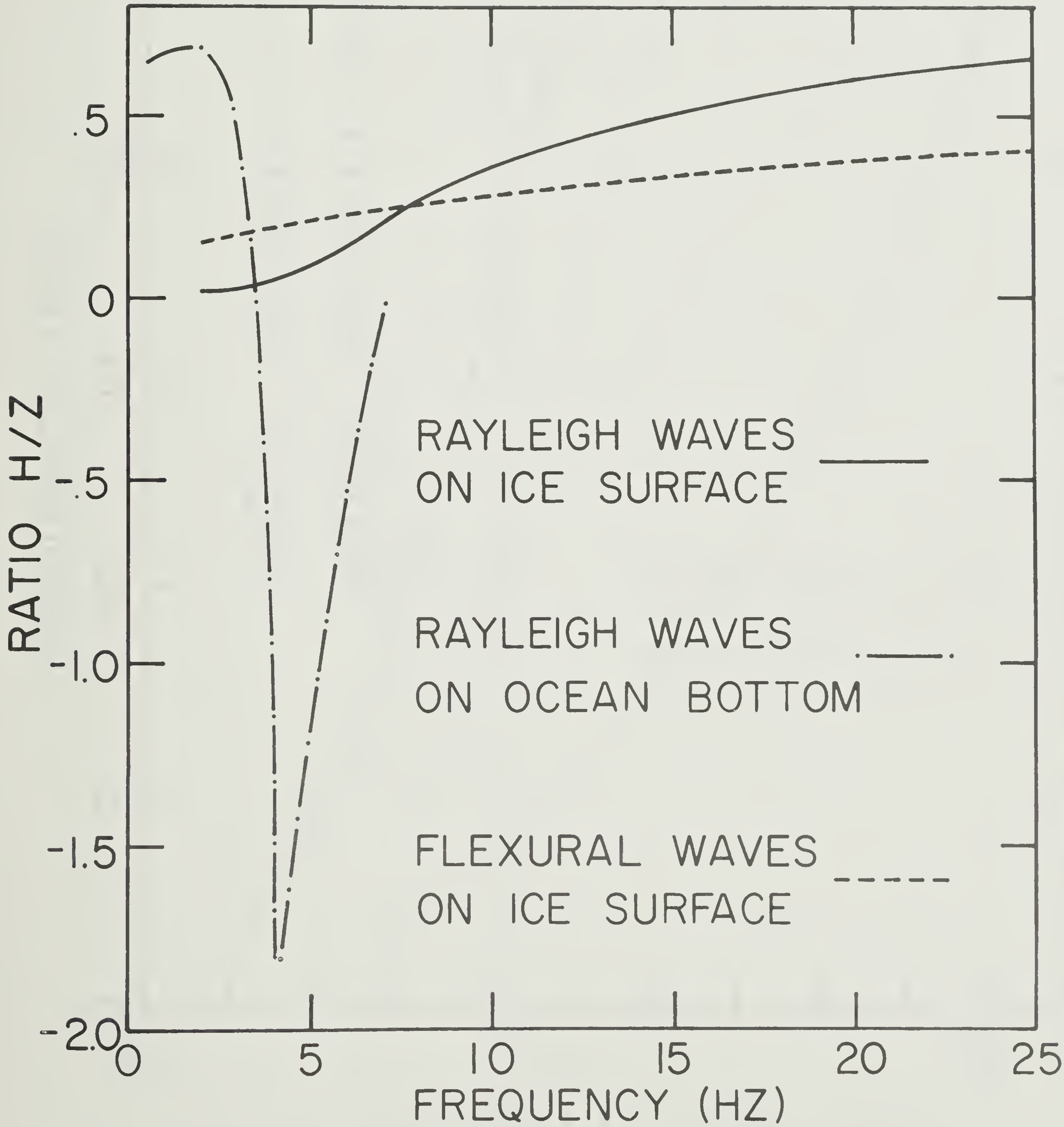

.5
0
-.5
-1.0
-1.5
-2.0
0
5
10
15
20
25
RATIO H/Z
FREQUENCY (HZ)
RAYLEIGH WAVES
ON ICE SURFACE
RAYLEIGH WAVES
ON OCEAN BOTTOM
FLEXURAL WAVES
ON ICE SURFACE

Fig. 5

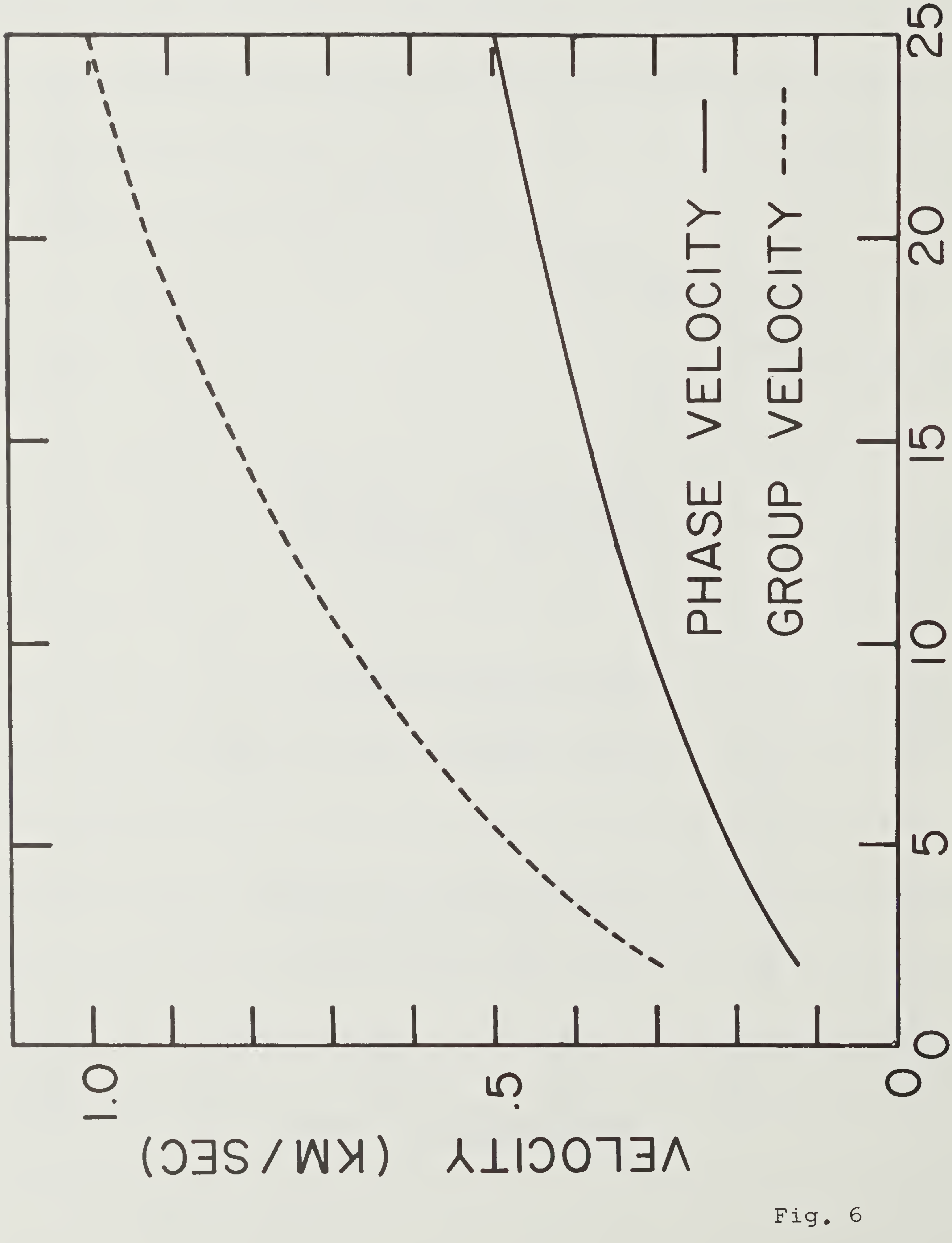

PHASE VELOCITY
GROUP VELOCITY
FREQUENCY (HZ)
0
5
10
15
20
25
VELOCITY (KM/SEC)
0
.5
1.0

Fig. 6

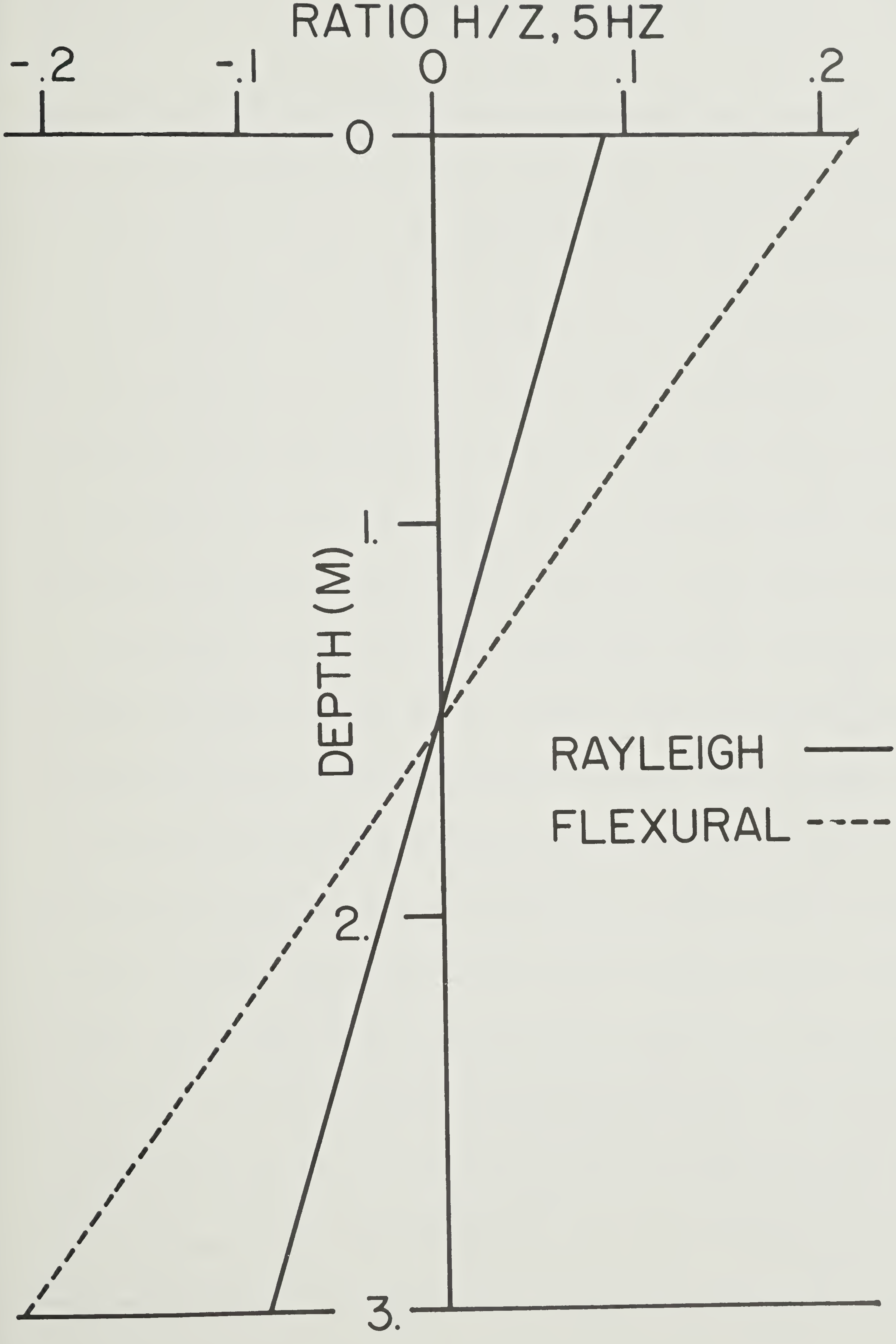


Fig. 7

shows the ratio of horizontal to vertical particle motion of the fundamental Rayleigh mode on the ice surface as well as on the ocean bottom. For comparison Figure 5 also shows the same ratio of flexural waves at the ice surface. Figure 6 shows the dispersion of the flexural waves for Model A. Flexural waves are surface waves traveling in the ice sheet with pressure decreasing exponentially with depth below the ice. These waves are weakly excited from sources at depth, but they form the predominant ice vibrations from pressure ridging and fracturing at the edges of the floe on which listening is done. Figure 7 shows that both Rayleigh waves and flexural waves exhibit prograde elliptical motion on the surface of the ice and retrograde motion on the bottom of the ice. Since the vertical motion is nearly constant in the ice from top to bottom for both flexural waves and Rayleigh waves, Figure 7 essentially shows the variation of horizontal motion with depth. Computations of the ratio of horizontal to vertical particle motion in the ice at frequencies up to 25 Hz exhibit a similar polarization to that of Figure 7 for both types of waves. On the basis of particle motion, the low frequency ice vibrations of the Rayleigh mode

Fig. 8

may be classified as antisymmetric vibrations of the plate; that is, a bending of the plate in a manner similar to that of flexural waves.

In Figure 8 the range dependence of pressure at 5 Hz is plotted for the fundamental Rayleigh mode. Source level, source depth and hydrophone depth are given in Table 2 for Model A.

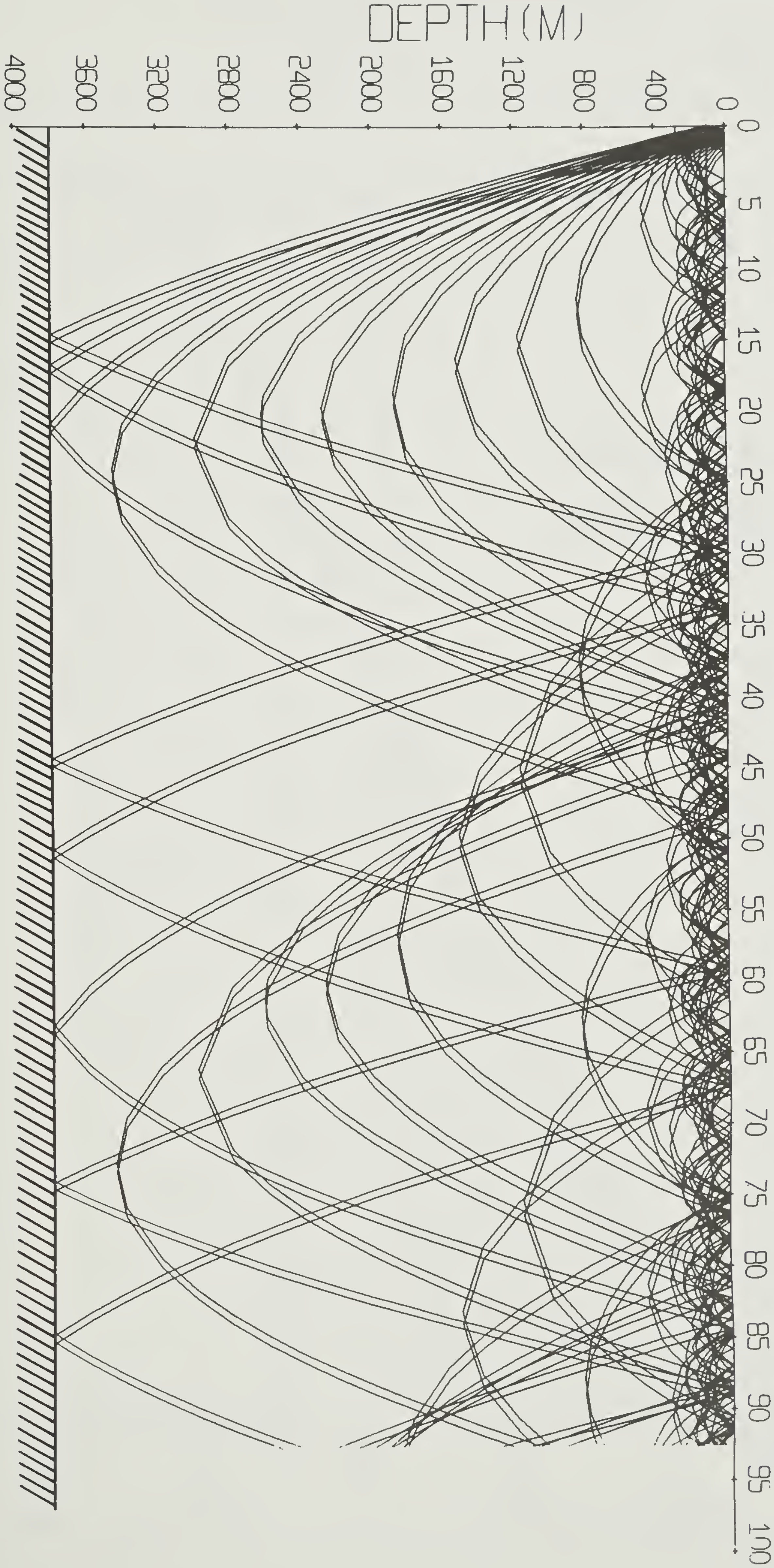

DEPTH(M)
0 400 800 1200 1600 2000 2400 2800 3200 3600 4000
RANGE(KM)
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

Fig. 9

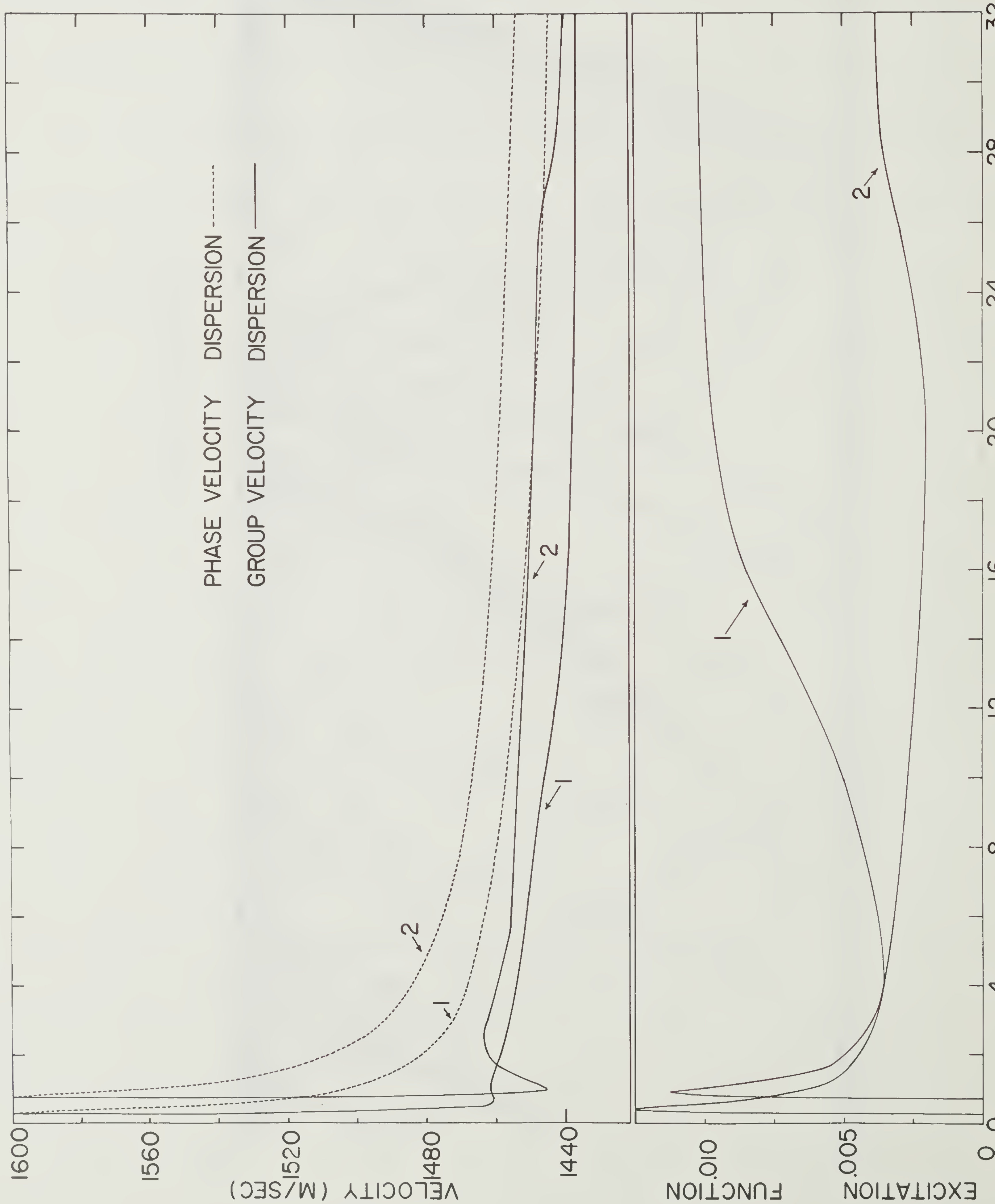

PHASE VELOCITY DISPERSION
GROUP VELOCITY DISPERSION
VELOCITY (M/SEC)
1600
1560
1520
1480
1440
EXCITATION FUNCTION
.010
.005
0
FREQUENCY (CPS)
0
4
8
12
16
20
24
28
32
1
2

Fig. 10

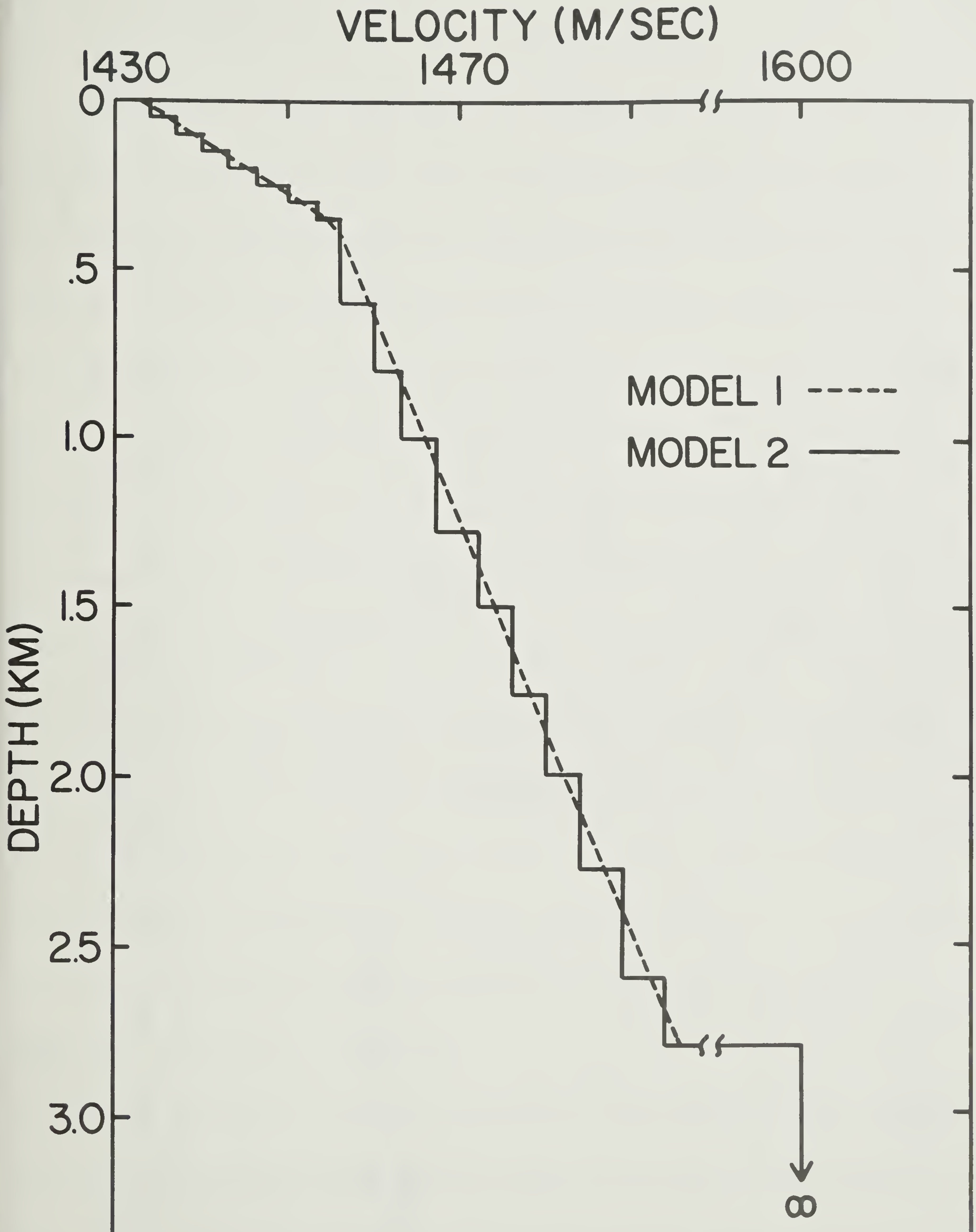

VELOCITY (M/SEC)
1430
1470
1600
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
DEPTH (KM)
MODEL 1
MODEL 2
∞

Fig. 11

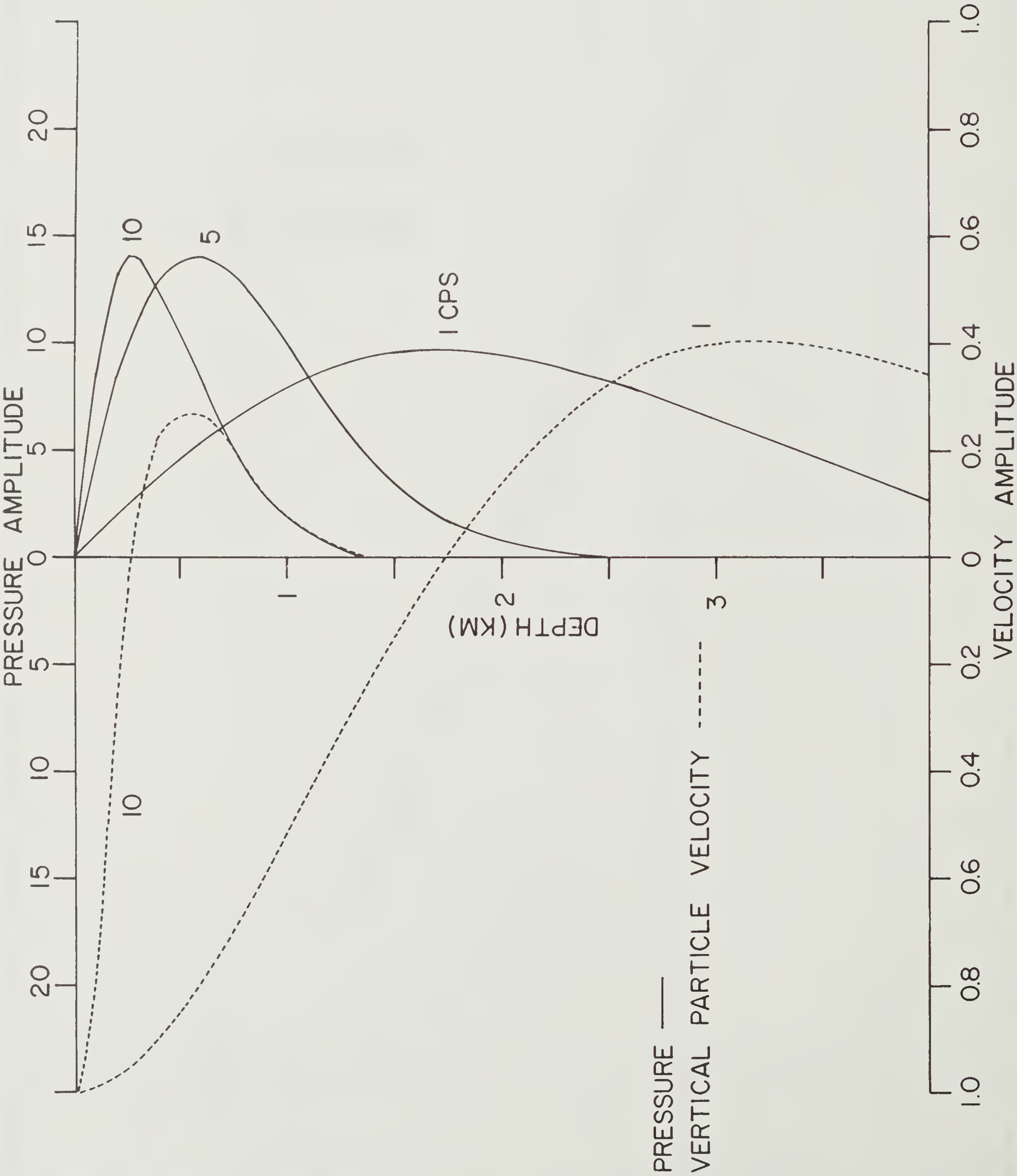
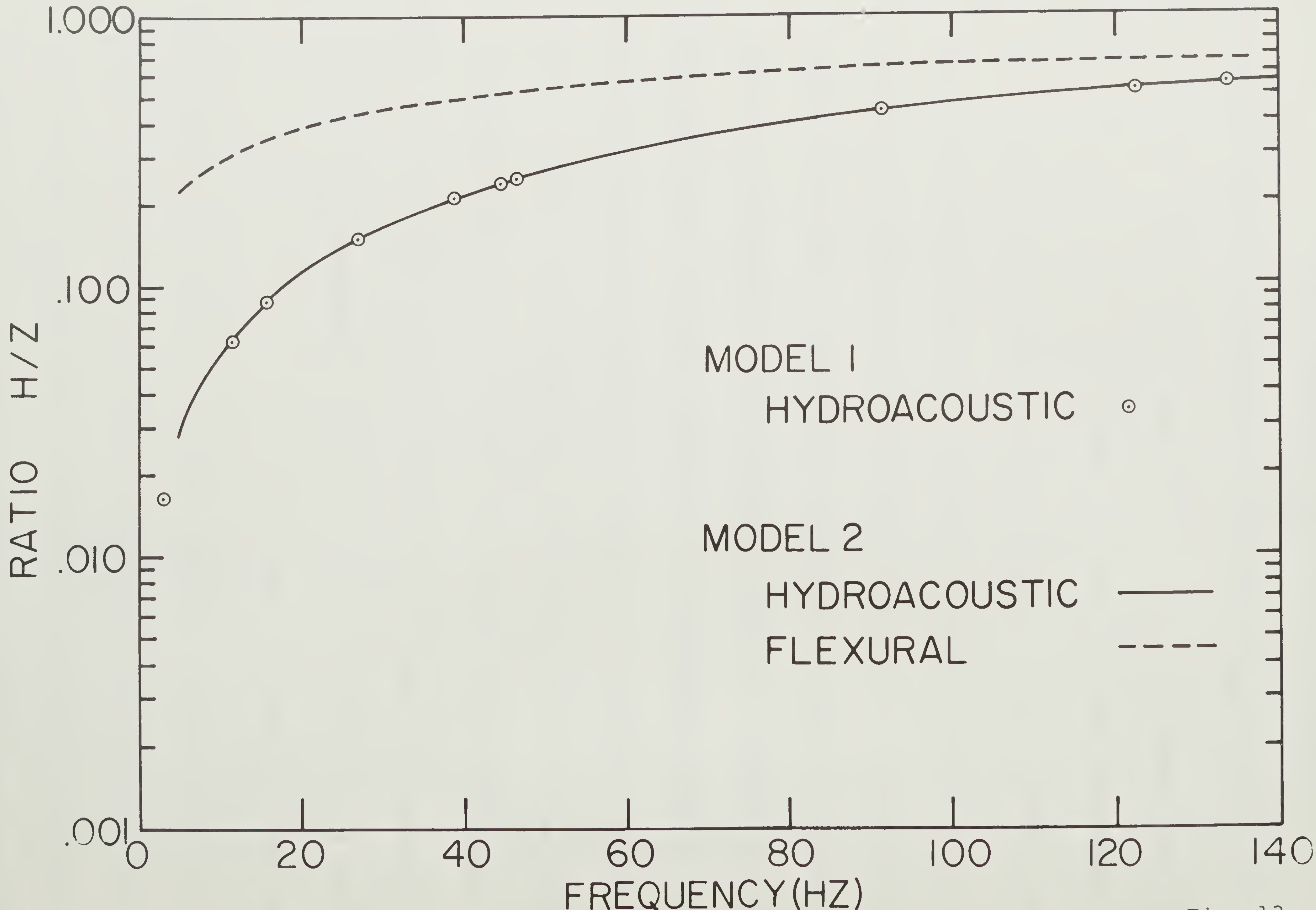

PRESSURE AMPLITUDE
20
15
10
5
0
5
10
15
20
10
5
1 CPS
1
10
DEPTH (KM)
1
2
3
VELOCITY AMPLITUDE
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
PRESSURE
VERTICAL PARTICLE VELOCITY

Fig. 12

SUMMARY OF WORK DONE ON BEHALF OF NOL

The present report completes a formulation for describing low-frequency propagation in the ice covered Arctic Ocean. In a report in preparation numerous numerical computations will be presented for deep and shallow water propagation to illustrate the effects of an ice layer on group and phase velocity dispersion, variation of pressure with depth in the ocean, particle motions in the ice, and the variation of pressure and particle motions as a function of range from the source. We summarize the principal features here in deep water for waves propagating in the SOFAR channel. Sample ray paths are shown in Figure 9. Figure 10 shows phase and group velocity dispersion and the depth independent excitation function for the first two modes computed for Model 2 of Figure 11 but in water 4KM deep. The variation of pressure and vertical particle velocity in the water for the first mode is shown in Figure 12. Parameters for the ice sheet are the same as in Table 1.

For an ice layer 3 m thick, typical of the central Arctic Ocean, the effect on pressure amplitudes at depth and dispersion is small, but even this thin ice sheet causes a large change in particle motions near the surface. The effect is similar to that of Rayleigh



1.000
.100
.010
.001
RATIO H/Z
0
20
40
60
80
100
120
140
FREQUENCY (HZ)
MODEL 1
HYDROACOUSTIC
MODEL 2
HYDROACOUSTIC
FLEXURAL

Fig. 13

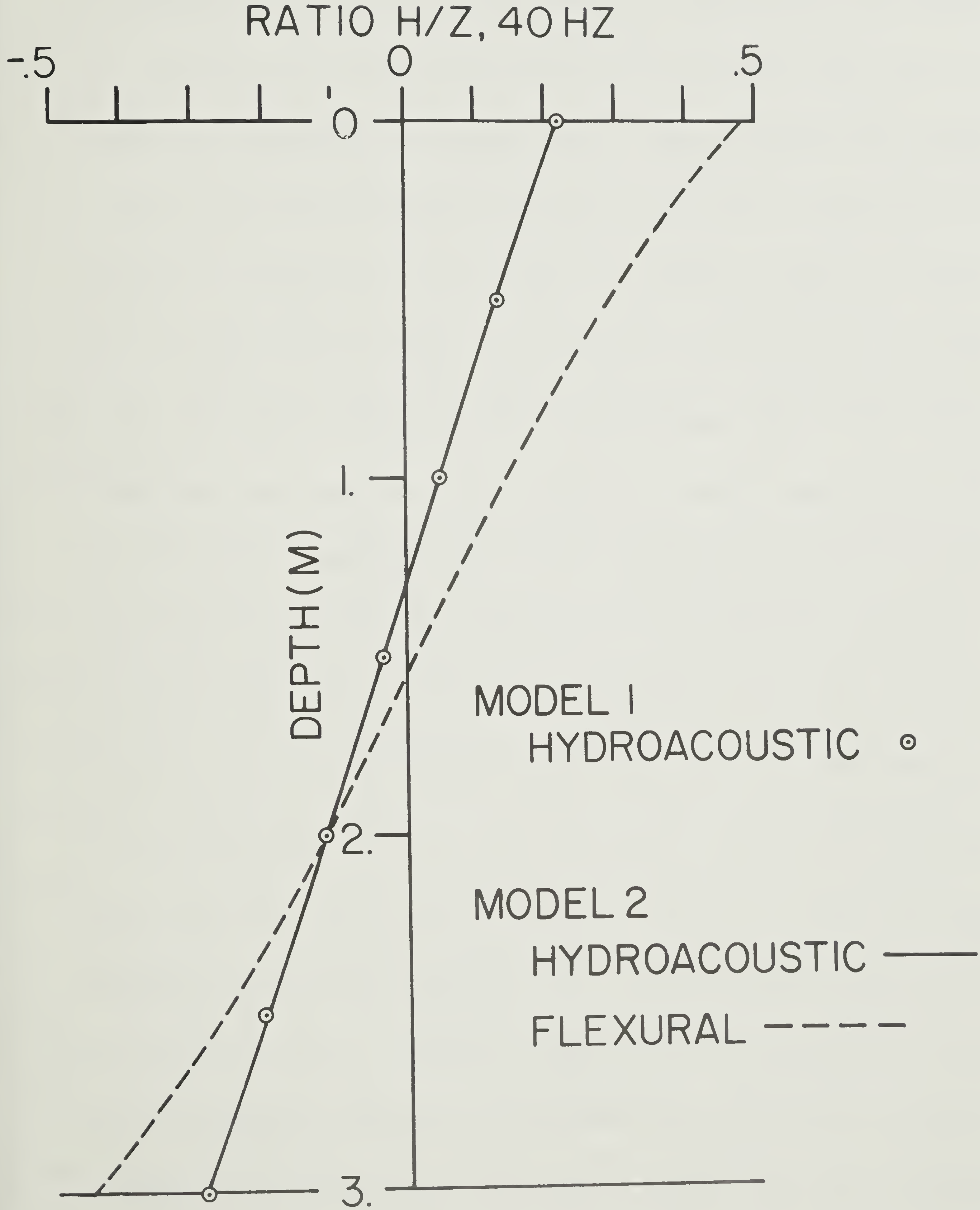

RATIO H/Z, 40 HZ
-.5
0
.5
0
1.
2.
3.
DEPTH (M)
MODEL 1
HYDROACOUSTIC
MODEL 2
HYDROACOUSTIC
FLEXURAL

Fig. 14

waves illustrated in the previous section. Waves are elliptically polarized in the plane of propagation. The ratio of horizontal to vertical particle motion is quite sensitive to ice thickness. To speed the numerical work, a rapid method was developed for computing particle in the ice by the W.K.B. method (Kutschale, 1972). Figure 13 shows an example of the ratio of horizontal to vertical particle motion at the ice surface for hydroacoustic waves of the first normal mode computed by the W.K.B. method (Model 1 of Fig. 11) and exact theory (Model 2 of Fig. 11). For comparison this same ratio is shown for flexural waves. Ice thickness is 3 m. A positive ratio corresponds to retrograde elliptical motion and a negative one to prograde elliptical motion. Flexural waves generated by large-scale ice movements at the boundaries of floes are the principal background noise in the ice and it is important to compare the particle motions of flexural waves in the ice with those of the hydroacoustic waves. Fig. 14 shows the principal features at low frequencies for both types of waves. Particle motion is retrograde elliptical at the surface and prograde elliptical at the bottom of the ice. Note that Rayleigh waves of Figures 5 and 7 also show this type of particle motion. Since the vertical motion

is nearly constant through the ice for both flexural waves and hydroacoustic waves, these plots essentially show the variation of horizontal particle motion with depth in the ice. It is significant that a node of horizontal particle motion often does not occur at the same depth in the ice for both hydroacoustic waves and flexural waves. This effect may be useful to improve the signal to noise ratio when horizontal component geophones are used as listening devices.

Future work will emphasize analysis of field data and comparison of these data with theory. Over one hundred recordings from explosions have been made in deep water and shallow water areas under various ice conditions employing as listening devices vertical and horizontal component geophones on the ice as well as hydrophones at depth. An analysis of these recordings will establish the effects of variations of ice thickness and roughness on acoustic wave velocities, wave amplitudes, and particle motions on the ice. Guided by these results and the numerical work additional, experiments will be made employing explosive and harmonic sources to verify the theoretical predictions on the variation of particle motion of hydroacoustic

and flexural waves as a function of depth in the ice and to determine whether in fact the signal to noise ratio can be enhanced by a horizontal component geophone buried near mid-depth in the ice.

Since large scale ice movements are the principal source of background noise we will investigate the spectral character of the ice vibrations generated by these movements. Data have been recorded from geophones and hydrophones installed in an active pressure ridge and additional experiments are planned.

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APPENDIX A
MATRIX ELEMENTS

Matrix elements (see Haskell 1953, and Dorman, 1962). Solid layers

$$(a_m)_{11} = (a_m)_{44} = \gamma_m \cos P_m - (\gamma_m - 1)\cos Q_m$$

$$(a_m)_{12} = (a_m)_{34} = i\left[r_{\alpha_m}^{-1}(\gamma_m - 1)\sin P_m + \gamma_m r_{\beta_m} \sin Q_m\right]$$

$$(a_m)_{13} = (a_m)_{24} = -(\rho_m c^2)^{-1}(\cos P_m - \cos Q_m)$$

$$(a_m)_{14} = i(\rho_m c^2)^{-1}\left[r_{\alpha_m}^{-1} \sin P_m + r_{\beta_m} \sin Q_m\right]$$

$$(a_m)_{21} = (a_m)_{43} = -i\left[\gamma_m r_{\alpha_m} \sin P_m + r_{\beta_m}^{-1}(\gamma_m - 1)\sin Q_m\right]$$

$$(a_m)_{22} = (a_m)_{33} = -(\gamma_m - 1)\cos P_m + \gamma_m \cos Q_m$$

$$(a_m)_{23} = i(\rho_m c^2)^{-1}(r_{\alpha_m} \sin P_m + r_{\alpha_m}^{-1} \sin Q_m)$$

$$(a_m)_{31} = (a_m)_{42} = \rho_m c^2 \gamma_m (\gamma_m - 1) \times (\cos P_m - \cos Q_m)$$

$$(a_m)_{32} = i\rho_m c^2 \left[r_{\alpha_m}^{-1} (\gamma_m - 1)^2 \sin P_m + \gamma_m^2 r_{\beta_m} \sin Q_m \right]$$

$$(a_m)_{41} = i\rho_m c^2 \left[\gamma_m^2 r_{\alpha_m} \sin P_m + r_{\beta_m}^{-1} (\gamma_m - 1)^2 \sin Q_m \right]$$

Liquid layers 4 x 4 matrices

$$(a_m)_{11} = (a_m)_{44} = 1$$

$$(a_m)_{12} = (a_m)_{13} = (a_m)_{14} = (a_m)_{21}$$

$$(a_m)_{24} = (a_m)_{31} = (a_m)_{34} = (a_m)_{41}$$

$$= (a_m)_{42} = (a_m)_{43} = 0$$

$$(a_m)_{22} = (a_m)_{33} = \cos P_m$$

$$(a_m)_{23} = \frac{i\, r_{\alpha_m} \sin P_m}{\rho_m c^2}$$

$$(a_m)_{32} = \frac{i\, \rho_m c^2 \sin P_m}{r_{\alpha_m}}$$

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The integral solution of the sound field is derived from a laminated interbedded liquid-solid half space. The last layer of infinite thickness is solid. The integral over wave number is conveniently transformed into the complex wave number plane yielding a sum of normal modes of propagation plus the sum of integrals along branch cuts. Waves corresponding to the normal modes predominate at ranges beyond several times the water depth and are considered in detail. Sample numerical computations are presented for propagation of Rayleigh waves in shallow water on the Arctic continental shelf.

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